Spreadsheet Use for Pipe Flow-Friction Factor Calculations

Course No: C03-022
Credit: 3 PDH

Harlan Bengtson, Ph.D., P.E.

Spreadsheets for Pipe Flow-Friction Factor Calculations

Harlan H. Bengtson, PhD, P.E.

COURSE CONTENT

1.   Introduction

Several kinds of pipe flow calculations can be made with the Darcy-Weisbach equation and the Moody friction factor. Many of the calculations require an iterative solution, so they are especially suitable for an Excel spreadsheet solution. This course includes discussion of the Darcy-Weisbach equation, the parameters in the equation along with the U.S. and S.I. units to be used, and the types of pipe flow calculations that are suitable for this equation. A spreadsheet that can be used to make Darcy-Weisbach/friction factor calculations is included with the course. Example calculations and discussion of the use of the course spreadsheet for making the calculations are also included.

Image Credit: Wikimedia Commons

2.   Learning Objectives

At the conclusion of this course, the student will

- Be able to calculate the Reynolds number for pipe flow with specified flow conditions

- Be able to determine whether a specified pipe flow is laminar or turbulent flow for specified flow conditions

- Be able to calculate the entrance length for pipe flow with specified flow conditions

- Be able to determine a value of the Moody friction factor from the Moody diagram, for given **Re** and ε/D.

- Be able to calculate a value of the Moody friction factor for given **Re** and ε/D, using the Moody friction factor equations.

- Be able to use the Darcy Weisbach equation and the Moody friction factor equations to calculate the frictional head loss and frictional pressure drop for a given flow rate of a specified fluid through a pipe with known diameter, length and roughness.

- Be able to use the Darcy Weisbach equation and the Moody friction factor equations to calculate the required diameter for a given flow rate of a specified fluid through a pipe with known length and roughness, with specified allowable head loss.

- Be able to use the Darcy Weisbach equation and the Moody friction factor equations to calculate the fluid flow rate through a pipe with known diameter, length and roughness, with specified frictional head loss.

- Be able to use the course spreadsheet to make pipe flow/friction factor calculations.

3.　Topics Covered in this Course

4.　Pipe Flow Background

The term pipe flow in this course is being taken to mean flow under pressure in a pipe, piping system, or closed conduit with a non-circular cross-section. Calculations for gravity flow in a circular pipe, such as a storm sewer, are done with open channel flow equations, and will not be discussed in this course.

The driving force for pressure pipe flow is typically pressure generated by a pump and/or flow from an elevated tank.

5.   Laminar and Turbulent Flow in Pipes

It is often useful to be able to determine whether a given pipe flow is laminar or turbulent. This is necessary, because different methods of analysis or different equations are sometimes needed for the two different flow regimes.

Laminar flow takes place for flow situations with low fluid velocity and high fluid viscosity. In laminar flow, all of the fluid velocity vectors line up in the direction of flow. Turbulent flow, on the other hand, is characterized by turbulence and mixing in the flow. It has point velocity vectors in all directions, but the overall flow is in one direction. Turbulent flow takes place in flow situations with high fluid velocity and low fluid viscosity. The figures below illustrate differences between laminar and turbulent flow in a pipe.

Laminar Flow (Re < 2100)

Turbulent Flow (Re > 4000)
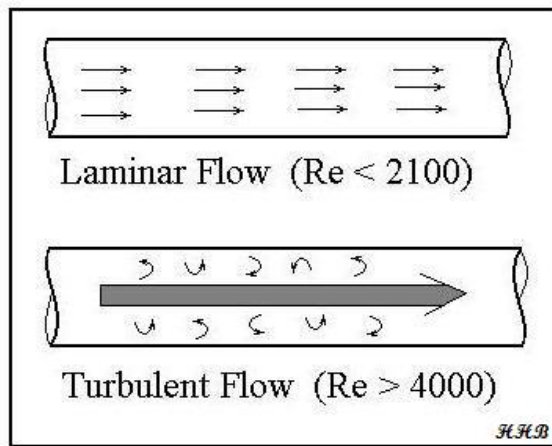
Laminar and Turbulent Pipe Flow

Laminar Pipe Flow

Turbulent Pipe Flow
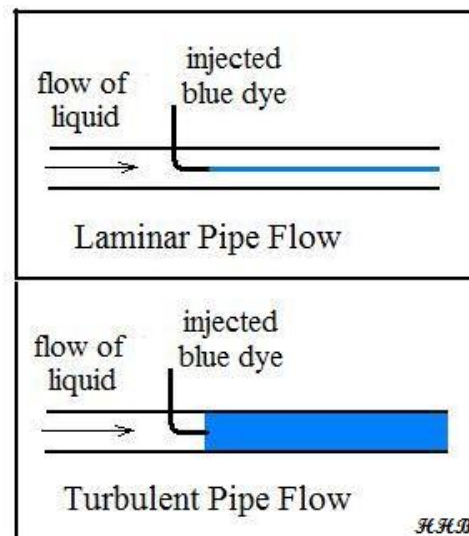
The figure at the right above illustrates the classic experiments of Osborne Reynolds, in which he injected dye into fluids flowing under a variety of conditions and identified the group of parameters now known as the Reynolds Number for determining whether pipe flow will be laminar or turbulent. He observed that under laminar flow conditions, the dye flows in a streamline and doesn't mix into the rest of the flowing fluid. Under turbulent flow conditions, the turbulence mixes dye into all of the flowing fluid. Based on Reynolds' experiments and subsequent measurements, the

criteria now in widespread use is that **pipe flow will be laminar for a Reynolds Number (Re) less than 2100 and it will be turbulent for a Re greater than 4000.** For $2100 < \text{Re} < 4000$, called the transition region, the flow may be either laminar or turbulent, depending upon factors like the entrance conditions into the pipe and the roughness of the pipe surface.

The Reynolds Number for flow in pipes is defined as: **Re = DVρ/μ,** where:

- **D** is the diameter of the pipe in ft (m for S.I.)

- **V** is the average fluid velocity in the pipe in ft/sec (m/s for S.I) (**V = Q/A**, where **Q** = volumetric flow rate and **A** = cross-sectional area of flow)

- ρ is the density of the fluid in slugs/ft^3 (kg/m^3 for S.I.)

- μ is the viscosity of the fluid in lb-sec/ft^2 (N-s/m^2 for S.I.)

Transport of water or air in a pipe is usually turbulent flow. Also pipe flow of other gases or liquids whose viscosity is similar to water will typically be turbulent flow. Laminar flow typically takes place with liquids of high viscosity, like lubricating oils.

Example #1: Water at 50°F is flowing at 0.6 cfs through a 4" diameter pipe. What is the Reynolds number of this flow? Is the flow laminar or turbulent?

Solution: From the table below the density and viscosity of water at 50°F are: ρ = 1.94 slugs/ft^3 and μ = 2.73 x 10^{-5} lb-sec/ft^2.

The water velocity, **V**, can be calculated from **V = Q/A = Q/(πD^2/4)** = $0.6/[\pi(1/3)^2/4]$ = 6.9 ft/sec.

Substituting values into **Re = DVρ/μ** gives:
Re = (1/3)(6.9)(1.94)/(2.73 x 10^{-5}), or **Re = 1.6 x 10^5**.

The Reynolds Number is greater than 4000, so **the flow is turbulent**.

The density and viscosity of the flowing fluid are often needed for pipe flow calculations. Values of density and viscosity of many fluids can be found in handbooks, textbooks and websites. Table 1 below gives values of density and viscosity for water over a range of temperatures from 32 ^{o}F to 70 ^{o}F.

Table 1. Density and Viscosity of Water

Temperature, °F	Density, slugs/ft³	Dynamic Viscosity, lb-s/ft²
32	1.940	3.732×10^{-5}
40	1.940	3.228×10^{-5}
50	1.940	2.730×10^{-5}
60	1.938	2.334×10^{-5}
70	1.936	2.037×10^{-5}

6. The Entrance Length and Fully Developed Flow

The Darcy Weisbach equation, which will be discussed in the next section, applies only to the fully developed portion of the pipe flow. If the pipe in question is long in comparison with its entrance length, then the entrance length effect is often neglected and the total length of the pipe is used for calculations. If the entrance length is a significant part of the total pipe length; however, then it may be necessary to make separate calculations for the entrance region.

The entrance region refers to the initial portion of the pipe in which the velocity profile is not "fully developed," as shown in the diagram below. At the entrance to the pipe, the flow is often constant over the pipe cross-section. In the region near the entrance, the flow in the center of the pipe is unaffected by the friction between the pipe wall and the fluid. At the end of the entrance region, the pattern of velocity across the pipe (the velocity profile) has reached its final shape.
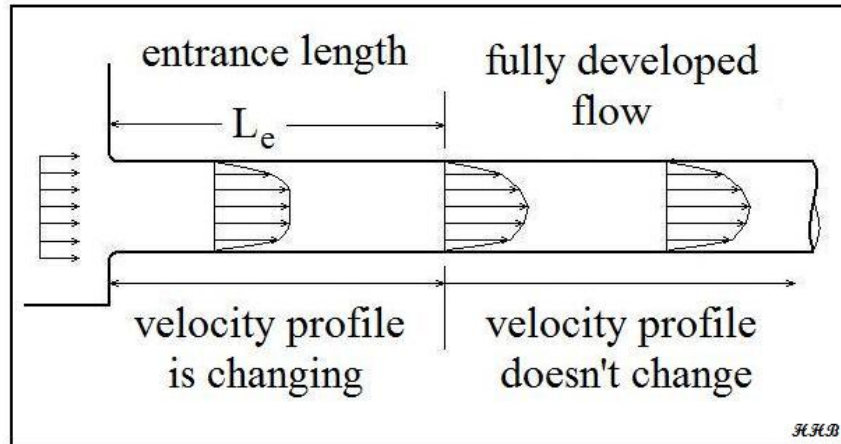
The entrance length, L_e, can be estimated if the Reynolds Number, Re, is known.

For turbulent flow (Re > 4000): **L_e/D = 4.4 Re$^{1/6}$.**

For laminar flow (Re < 2100): **L_e/D = 0.06 Re.**

Example #2: Estimate the entrance length for the water flow described in example #1: (0.6 cfs of water at 50°F, flowing through a 4" diameter pipe).

Solution: From Example # 1, **Re** = 1.6 x 10^5. Since Re > 4000, this is turbulent flow, and **L_e/D = L_e/(1/3)** = 4.4(1.6 x 10^5)$^{1/6}$ = 32.4
Thus L_e = (1/3)(32.4) = **10.8 ft = L_e**

7. The Darcy Weisbach Equation

The **Darcy Weisbach equation** is a widely used empirical relationship among several pipe flow variables. The equation is: **h_L = f(L/D)(V^2/2g),** where:

- **L** = pipe length, ft (m for S.I. units)

- **D** = pipe diameter, ft (m for S.I. units)

- **V** = average velocity of fluid (= Q/A), ft/sec (m/s for S.I. units)

- **h_L** = frictional head loss due to flow at an average velocity, V, through a pipe of diameter, D, and length, L, ft (ft-lb/lb) (m or N-m/N for S.I. units).

- **g** = acceleration due to gravity = 32.2 ft/sec^2 (9.81 m/s^2)

- **f** = Moody friction factor (a dimensionless empirical factor that is a function of Reynolds Number and ε/D, where:

- **ε** = an empirical pipe roughness, ft (m or mm for S.I. units)

NOTE: Although the Darcy Weisbach equation is an empirical equation, it is also a dimensionally consistent equation. That is, there are no dimensional constants in the equation, so any consistent set of units that gives the same dimensions on both sides of the equation can be used. A typical set of U.S. units and a typical set of S.I. units are shown in the list above.

Table 2, below, shows some typical values of pipe roughness for common pipe materials, drawn from information on several websites.

Table 2. Pipe Roughness Values

Pipe Material	Roughness, ε
	feet
drawn brass or copper	0.000005
PVC pipe	0.000005
commercial steel	0.000150
wrought iron	0.000150
asphalted cast iron	0.000400
galvanized iron	0.000500
cast iron	0.000850
concrete	0.001 - 0.01

Frictional pressure drop for pipe flow is related to the frictional head loss through the equation: $\Delta P_f = \rho g h_L = \gamma h_L$, where:

- h_L = frictional head loss (ft or m) as defined above

- ρ = fluid density, slugs/ft^3 (kg/m^3 for S.I.)

- g = acceleration due to gravity = 32.17 ft/sec^2 = 9.81m/s^2

- γ = specific weight, lb/ft^3 (N/m^3 for S.I.)

8.   Obtaining a Value for the Friction Factor

A value of the Moody friction factor, **f**, is needed for any calculations with the Darcy Weisbach equation other than empirical determination of the friction factor by measuring all of the other parameters in the equation. A common method of obtaining a value for **f** is graphically, from the Moody friction factor diagram, first presented by L. F. Moody in his classic 1944 paper in the *Transactions of the ASME*. (Ref. #1).   The Moody friction factor diagram, shown in the diagram below, is now available in many handbooks and textbooks and on many websites.
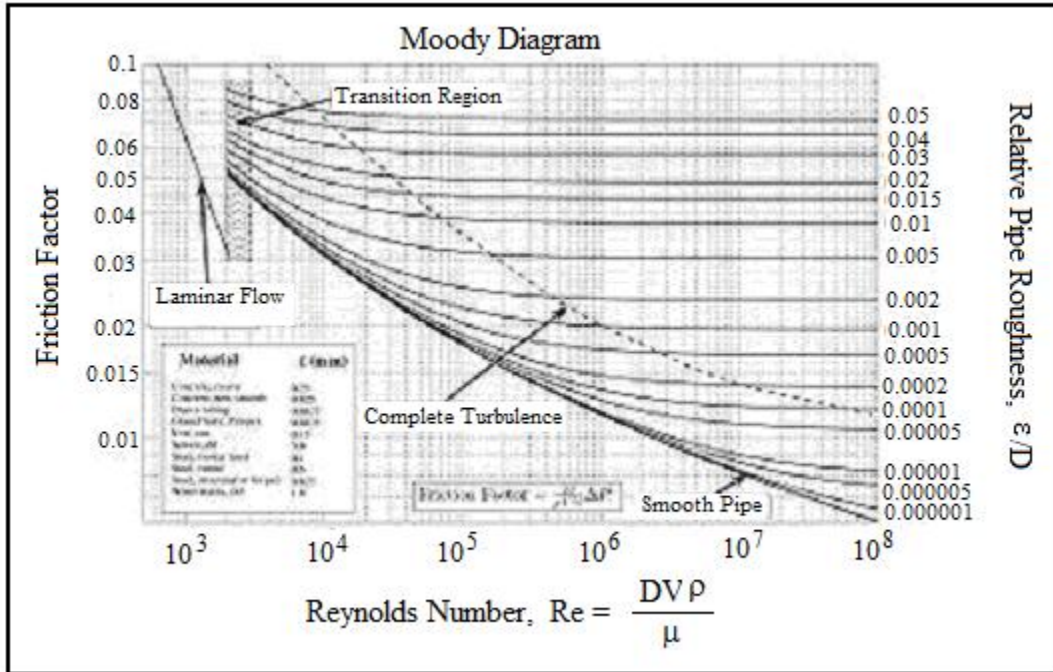
Source: http://www.thefullwiki.org/Moody_diagram

When using Excel spreadsheets for pipe flow calculations with the Darcy Weisbach equation, it is more convenient to use equations for the Moody friction factor, **f**, rather than a graph like the Moody diagram. There are indeed equations available that give the relationships between Moody friction factor and **Re** & ε/**D** for four different portions of the Moody diagram. The four portions of the Moody diagram are:

i)   **laminar flow** (**Re** < 2100 – the straight line at the left side of the Moody diagram);

ii)   **smooth pipe turbulent flow** (the dark curve labeled "smooth pipe" in the Moody diagram – **f** is a function of **Re** only in this region);

iii) **completely turbulent region** (the portion of the diagram above and to the right of the dashed line labeled "complete turbulence" – **f** is a function of ε/**D** only in this region);

iv) **transition region** (the portion of the diagram between the "smooth pipe" solid line and the "complete turbulence" dashed line – **f** is a function of both **Re** and **ε/D** in this region).

The equations for these four regions are shown in the box below:

$$\text{Laminar Flow:} \quad f = \frac{64}{Re}$$

$$\text{Smooth Pipe Turbulent Flow:} \quad f = \frac{0.316}{Re^{1/4}}$$

$$\text{Completely Turbulent Flow:} \quad f = \left[1.14 + 2 \log_{10} \left(\frac{D}{\varepsilon} \right) \right]^{-2}$$

$$\text{Transition Region:} \quad f = \left\{ -2 \log_{10} \left[\frac{(\varepsilon/D)}{3.7} + \frac{2.51}{Re\,(f^{1/2})} \right] \right\}^{-2}$$

Example #3: Calculate the value of the Moody friction factor for pipe flow with **Re** = 10^7 and **ε/D** = 0.005.

Solution: From the Moody diagram above, it is clear that the point, **Re** = 10^7 and **ε/D** = 0.005, is in the "complete turbulence" region of the diagram. Thus:

$$\mathbf{f} = [1.14 + 2 \log_{10}(D/\varepsilon)]^{-2} = [1.14 + 2 \log_{10}(1/0.005)]^{-2} = \mathbf{0.0303 = f}$$

A Spreadsheet as a Friction Factor Calculator – The use of a spreadsheet is an attractive alternative to the Moody diagram for determining a value of the Moody friction factor for specified values of Reynolds number, **Re**, and roughness ratio, **ε/D**. The screenshot diagram below shows how the friction factor calculation is implemented in the course spreadsheet, using the equations for f shown above. This screenshot is the top part of the worksheet at the "head loss" tab. It shows the solution to Example #4 below. The friction factor calculation is very similar for the other two tabs.

Instructions: *Enter values in blue boxes. Spreadsheet calculates values in yellow boxes*

1. Determine Friction Factor, f, assuming completely turbulent flow $\{f = [1.14 + 2 \log_{10}(D/\varepsilon)]^{-2}\}$

Inputs (enter values in the blue boxes)			**Calculations** (done by spreadsheet)	
Pipe Diameter, D_{in} =	6	in	Pipe Diameter, D =	0.5000	ft
Pipe Roughness, ε =	0.0005	ft	Friction Factor, f =	0.01962	
Pipe Length, L =	100	ft	Cross-Sect. Area, A =	0.1963	ft^2
Pipe Flow Rate, Q =	0.600	cfs	Ave. Velocity, V =	3.1	ft/sec
Fluid Density, ρ =	1.94	slugs/ft^3	Reynolds number, Re =	108,575	
Fluid Viscosity, μ =	0.0000273	lb-sec/ft^2			

2. Check on whether the given flow is "completely turbulent flow"

(Calculate f with the transition region equation and see if differs from the one calculated above.)

$$f = \{-2 \cdot \log_{10}[((\varepsilon/D)/3.7)+(2.51/(Re \cdot (f^{1/2})))]\}^{-2}$$

Transition Region Friction Factor, f:	f =	0.0221	
Repeat calc of f using new value of f:	f =	0.0220	
Repeat again if necessary:	f =	0.0220	

Example #4:   Use the course spreadsheet to find the value of the Moody friction factor for a flow of 0.60 cfs of water at 50°F through a 6 inch diameter pipe with surface roughness, $\varepsilon = 0.0005$.

Solution: The top part of the spreadsheet shown above has provision for user input of pipe diameter, **D**; pipe wall roughness, ε; pipe length, **L**; pipe flow rate, **Q**; fluid density, ρ; and fluid viscosity, μ. The spreadsheet is set up to then calculate the Moody friction factor, **f**, with the equation, $f = [1.14 + 2 \log_{10}(D/\varepsilon)^{-2}]$, which is for completely turbulent flow. After obtaining an initial estimate for **f** with this equation, step 2 in the spreadsheet uses an iterative process to recalculate **f** using the most general equation:

$$f = \{-2*\log_{10}[((\varepsilon/D)/3.7) + (2.51/(Re*(f^{1/2})))]\}^{-2}$$

This calculation uses the initial estimate of **f** to get a new estimate. The spreadsheet has provision for repeating this process a couple more times until succeeding values of **f** are the same.

As shown in the screenshot above, putting the given values in the indicated input cells of the spreadsheet leads to the result: **f = 0.0220**

9. Calculation of Frictional Head Loss/Pressure Drop

A. Straight Pipe Head Loss

Calculation of the frictional head loss for a specified flow rate of a given fluid at a given temperature through a pipe of known diameter, length, and material can be done using the Darcy Weisbach equation [$h_L = f(L/D)(V^2/2g)$]. The step-by-step process for this calculation is as follows:

a. Determine the density and viscosity of the flowing fluid at the specified temperature

b. Calculate the fluid velocity from $V = Q/A = Q/(\pi D^2/4)$

c. Calculate the Reynolds Number ($Re = DV\rho/\mu$)

d. Find the pipe roughness value (ε) for the specified pipe material

e. Calculate the pipe roughness ratio, (ε/D)

f. Determine the Moody friction factor value using the Moody diagram and/or friction factor equation(s), with the calculated values of **Re** and **ε/D**.

g. Calculate the frictional head loss using the Darcy Weisbach equation and the specified or calculated values of **L**, **D**, and **V**.

h. If desired calculate the frictional pressure drop

Example #5: Calculate the frictional head loss in ft and the frictional pressure drop in psi, for a flow rate of 0.60 cfs of water at 50°F, through 100 ft of 6 inch diameter wrought iron pipe. Use the Moody diagram or the course spreadsheet to get the value for **f**.

Solution: Following the steps outlined above:

a. At 50°F, the needed properties of water are:
   density $= \rho = 1.94$ slugs/ft^3, viscosity $= \mu = 2.73$ x 10^{-5} lb-s/ft^2

b. Water velocity $= \mathbf{V} = \mathbf{Q/(\pi D^2/4)} = 0.60/(\pi(6/12)^2/4) = 3.1$ ft/sec

c. Reynolds number $= \mathbf{Re} = \mathbf{DV\rho/\mu} =$
   $(0.5)(3.1)(1.94)/(2.73$ x $10^{-5}) = 1.08$ x 10^5

d. From the pipe roughness table on page 8, for wrought Iron:
   $\varepsilon = 0.0005$ ft

e. Pipe roughness ratio $= \mathbf{\varepsilon/D} = 0.0005/0.5 = 0.001$

f. From the Moody diagram, the point for $\mathbf{Re} = 1.08$ x 10^5 and $\mathbf{\varepsilon/D} = 0.001$, is in the transition region and the friction factor, $\mathbf{f}$, is approximately: $\mathbf{f} = 0.02$

g. Substituting the given values: $\mathbf{D} = 6$ inches $= 0.5$ ft and $\mathbf{L} = 100$ ft, along with the calculated values: $\mathbf{V} = 3.1$ ft/sec and $\mathbf{f} = 0.02$, and the acceleration due to gravity, $\mathbf{g} = 32.2$ ft/sec^2, the Darcy Weisbach equation becomes: $\mathbf{h_L = f(L/D)(V^2/2g)}$
   $= (0.02)(100/0.5)[3.1^2/(2*32.2)] = \mathbf{0.58}$ **ft** $= \mathbf{h_L}$

h. The frictional pressure drop is $\mathbf{\Delta P_f = \rho g h_L} = 1.94*32.2*0.58$
   $= 36$ lb/ft$^2 = 36/144$ psi $= \mathbf{0.25}$ **psi** $= \mathbf{\Delta P_f}$

Example #6: Repeat the calculations in Example #4, for the frictional head loss in ft and the frictional pressure drop in psi, for a flow rate of 0.60 cfs of water at 50°F, through 100 ft of 6 inch diameter wrought iron pipe, using the course spreadsheet.

Solution: The image on page 18 shows a screenshot of the worksheet at the "head loss" tab in the course spreadsheet, with the solution to this example.

Step 1 in the solution consists of entering a value for each of the following inputs in the column on the left at the upper part of the spreadsheet:

- the pipe diameter, D_{in}, in inches;

- the pipe roughness, ε, in ft;

- the pipe length, L, in ft;

- the pipe flow rate, Q, in cfs;

- the fluid density, ρ, in slugs/ft^3; and

- the fluid viscosity, μ, in lb-sec/ft^2.

When all the required input values are entered, the spreadsheet will calculate the parameters shown in the "calculations" column at the right side of the spreadsheet:

- pipe diameter, D, in ft, which is needed to calculate:

- Moody friction factor using $f = [1.14 + 2\log_{10}(D/\varepsilon)]^{-2}$ (That is, assuming the flow is in the "completely turbulent flow" region.)

- cross-sectional area of flow in the pipe, A, in ft^2, which is needed to calculate:

- average velocity of the fluid in the pipe, V, in ft/sec

- Reynolds number, Re

Step 2 in the spreadsheet is the iterative calculation of the Moody friction factor, f, using the transition region equation:

$$f = \{-2\log_{10}[((\varepsilon/D)/3.7) + (2.51/(Re\ f^{1/2}))]\}^{-2},\ \text{as follows:}$$

a. Calculate **f** with the transition region equation, using the **Re** value just calculated and the **f** value calculated assuming completely turbulent flow.

b. If this calculated value of **f** is different than the one calculated with the completely turbulent flow assumption, then the flow **is** in transition region and **f** should be calculated again with the transition region equation, using the last calculated value of **f** in the right hand side of the equation.

c. This is repeated until two subsequent calculations give the same value of **f**.

Step 3 in the spreadsheet is calculation of h_L using the Darcy Weisbach equation and calculation of ΔP_f using $\Delta P_f = \rho g h_L$. The calculated frictional head loss and frictional pressure drop, as shown in the spreadsheet image are:

$$h_L\ =\ 0.64\ \text{ft} \quad \text{and} \quad \Delta P_f\ =\ 0.28\ \text{psi}$$

These values were calculated using the final value of **f** calculated in the spreadsheet: **f** = 0.0220.

For comparison the value of **f** = 0.020 estimated from the Moody diagram led to these values:

$$h_L\ =\ 0.58\ \text{ft} \quad \text{and} \quad \Delta P_f\ =\ 0.25\ \text{psi}$$

The values calculated by the Excel spreadsheet using the Moody friction factor equations are the best estimates for h_L and ΔP_f. The value of **f** from the Moody diagram did, however give values of h_L and ΔP_f close to those from the equations. A larger, better Moody diagram would allow a more precise estimate of **f**.

Pipe Flow/Friction Factor Calculations I: (U.S. units)

Calculation of Head Loss, h_L, and Frictional Pressure Drop, ΔP_f,

for given flow rate, Q, pipe diam., D, pipe length, L,

pipe roughness, ε, and fluid properties, ρ & μ.

Instructions: *Enter values in blue boxes. Spreadsheet calculates values in yellow boxes*

1. Determine Friction Factor, f, assuming completely turbulent flow $\{f = [1.14 + 2 \log_{10}(D/\varepsilon)]^{-2}\}$

Inputs (enter values in the blue boxes)   **Calculations** (done by spreadsheet)

Inputs			Calculations		
Pipe Diameter, D_{in} =	6	in	Pipe Diameter, D =	0.5000	ft
Pipe Roughness, ε =	0.0005	ft	Friction Factor, f =	0.01962	
Pipe Length, L =	100	ft	Cross-Sect. Area, A =	0.1963	ft^2
Pipe Flow Rate, Q =	0.600	cfs	Ave. Velocity, V =	3.1	ft/sec
Fluid Density, ρ =	1.94	slugs/ft^3	Reynolds number, Re =	108,575	
Fluid Viscosity, μ =	0.0000273	lb-sec/ft^2			

2. Check on whether the given flow is "completely turbulent flow"

(Calculate f with the transition region equation and see if differs from the one calculated above.)

$f = \{-2 \log_{10}[((\varepsilon/D)/3.7)+(2.51/(Re^*(f^{1/2})))]\}^{-2}$

Transition Region Friction Factor, f:	f =	0.0221
Repeat calc of f using new value of f:	f =	0.0220
Repeat again if necessary:	f =	0.0220

3. Calculate h_L and ΔP_f, using the final value for f calculated in step 2

Equations: $h_L = f(L/D)(V^2/2g)$ and $\Delta P_f = \rho g h_L$

Frictional Head Loss, h_L =	0.64	ft
Frictional Pressure Drop, ΔP_f =	40	psf
Frictional Pressure Drop, ΔP_f =	0.28	psi

B. Minor Losses

The term "minor losses" refers to head loss or pressure drop due to flow through pipe fittings (e.g. valves, elbows, or tees), entrances and exits, changes in pipe diameter, and any other causes of head loss besides the straight pipe portion of the flow.

The general equation for calculating the minor loss of a fitting in a pipe system is: $h_L = K(V^2/2g)$, where **K** is the minor loss coefficient for the particular fitting. For several fittings in a pipe system, all of the **K** values can be summed to give: $h_L = \Sigma K(V^2/2g)$.

Values of **K** for fittings, changes in cross-section, etc are available in many engineering handbooks and textbooks and on internet sites. Table 3, below gives typically used values of **K** for some common pipe fittings.

Table 3. Minor Loss Coefficients for Pipe Fittings

Fitting	Loss Coefficient, K
Globe valve, fully open	10.0
Angle valve, fully open	5.0
Swing check valve, fully open	2.5
Gate valve, fully open	0.2
Short-radius elbow	0.9
Medium-radius elbow	0.8
Long-radius elbow	0.6
45 degree elbow	0.4
Close return bend	2.2
Standard tee, flow through run	0.6
Standard tee, flow through branch	1.8
Square entrance	0.5
Exit	1.0

Source: U.S. EPA, EPANET2 User Manual,
http://www.epa.gov/nrmrl/wswrd/dw/epanet/EN2manual.PDF

Example #7: What is the total head loss due to minor losses for the 0.60 cfs of water at 50°F flowing through 100 ft of 6 inch diameter pipe as in Examples #4 and #5, if the piping system contains one fully open check valve, three medium radius elbows, and one standard tee with flow through the branch.

Solution: The sum of the minor loss coefficients is:

$\Sigma K = 2.5 + (3)(0.8) + 1.8 \ = \ 6.7$

$h_L = \Sigma K(V^2/2g) \ = \ (6.7)[3.1^2/(2*32.2)] \ = \ \textbf{1.0 ft} \ = \ h_L \ \textbf{(minor losses)}$

10.   Calculation of Required Pipe Diameter

Calculation of the required pipe diameter for a specified flow rate of a given fluid at a given temperature through a pipe of known length and material, with a specified allowable head loss can be done using the Darcy Weisbach equation. The step-by-step process for this calculation is as follows:

a. Determine the density, ρ, and viscosity, μ, of the flowing fluid at the specified temperature.

b. Obtain a value of pipe roughness, ϵ, for the specified pipe material.

c. Identify the known values for frictional head loss, h_L, flow rate, Q, and pipe length, L.

d. Select an assumed pipe diameter, D, to use as a starting point.

   NOTE: Standard U.S. pipe sizes are given at the bottom of the spreadsheet image on page 21.

e. Using the pipe roughness, ϵ, and the assumed pipe diameter, D, calculate the Moody friction factor, f, assuming completely turbulent flow.

f. Use the assumed pipe diameter to calculate the pipe cross-sectional area, **A**, and use it together with the specified flow rate through the pipe, to calculate the average fluid velocity in the pipe, **V**.

g. Calculate the Reynolds number: $\mathbf{Re = DV\rho/\mu.}$

h. Use the calculated values of **Re** and **f** to recalculate **f** using the transition region equation (which gives **f** as a function of **Re** and **f**).

i. Use the following iterative process to calculate the Moody friction factor, **f**: If the value of **f** calculated in step h is different from that calculated in step e, repeat step h using the most recent value calculated for **f**. Repeat as many times as necessary until two subsequent calculations give the same value of **f**.

j. Using known values for $\mathbf{h_L}$ and **L**, and calculated values for **f** and **V**, calculate pipe diameter **D** with the Darcy Weisbach equation.

k. Use the following iterative process to calculate the pipe diameter, **D**:

- If the value of **D** calculated in step j is larger than the assumed value, replace the assumed value of **D** with a larger standard pipe diameter and recalculate **D** (steps e through j).

- If the value of **D** calculated in step j is smaller than the assumed value, replace the assumed value of **D** with a smaller standard pipe diameter and recalculate **D** (steps e through j).

- Repeat as many times as necessary to find the smallest standard pipe diameter for which the next calculated value of **D** is smaller than the assumed value. **This is the required value of D.**

Example #8: Use an Excel spreadsheet to calculate the required pipe diameter to carry 0.60 cfs of water at 50°F, through 100 ft of galvanized pipe, with an allowable head loss of 20 ft.

Pipe Flow/Friction Factor Calculations III: (U.S. units)

Calculation of pipe diameter, D, for given flow rate, Q, pipe length, L, pipe roughness, ε, head loss, h_L, and fluid properties, ρ & μ.

(NOTE: This is an iterative calculation. An initial assumed value of D will be used.)

Instructions: *Enter values in blue boxes. Spreadsheet calculates values in yellow boxes*

1. Determine Friction Factor, f, assuming completely turbulent flow $\{f = [1.14 + 2\,\log_{10}(D/\varepsilon)]^{-2}\}$

Inputs

				Calculations		
Allowable Head Loss, h_L =	20	ft		Assumed Pipe Diam, D_{in}* =	4	in
Pipe Roughness, ε =	0.0005	ft		Pipe Diameter, D =	0.3333	ft
Pipe Length, L =	100	ft		Friction Factor, f =	0.02170	
Pipe Flow Rate, Q =	0.600	cfs		Cross-Sect. Area, A =	0.0873	ft²
Fluid Density, ρ =	1.94	slugs/ft³		Ave. Velocity, V =	6.9	ft/sec
Fluid Viscosity, μ =	0.000027	lb-sec/ft²		Reynolds number, Re =	164,672	

2. Check on whether the given flow is "completely turbulent flow"

(Calculate f with the transition region equation and see if differs from the one calculated above.)

$[\,f = \{-2^*\log_{10}[((\varepsilon/D)/3.7)+(2.51/(Re^*(f^{1/2}))]\}^{-2}\,]$

Transistion Region Friction Factor, f:	f =	0.0230
Repeat calc of f using new value of f:	f =	0.0230
Repeat again if necessary:	f =	0.0230

3. Calculate pipe diameter, D using the final value for f calculated in step 2

$$[\, D \,=\, f(L/h_L)(V^2/2g)\,]$$

| Pipe Diameter, D = | 0.0845 | ft | = | 1.0 | in |

NOTE: This iterative procedure doesn't converge smoothly to a solution. If the calculated pipe diameter in this step is larger than the assumed pipe diameter above, then replace the assumed pipe diameter value with the next larger standard pipe size. Repeat until you find the smallest standard pipe diameter that gives a smaller calculated required pipe diameter. That is your solution.

With the example values given here, an assumed pipe diameter of 3 inches gives a calculated pipe diameter requirement of 3.4 inches, increasing the assumed pipe diameter to the next standard pipe size (3.5 inches) gives a calculated pipe diameter requirement of 1.8 inches, so a 3.5 inch size is the minimum standard pipe diameter that will do the job.

*Standard nominal U.S. pipe sizes in inches:

1/8, 1/4, 3/8, 1/2, 3/4, 1, 1 1/4, 1 1/2, 2, 2 1/2, 3, 3 1/2, 4, 5, 6, 8, 10, 12, 14, 16, 18, 20, 24, 30, 42, 48, 54, 60

Solution: The Excel spreadsheet in the two images above came from the "diameter" tab in the course spreadsheet and shows the solution to this example. The spreadsheet calculations correlate with the steps a through k outlined above for this type of calculation.

- Steps a, b, & c consist of entering values for the required inputs, h_L, ε, **L, Q, ρ**, and **μ** in the column on the left in the upper part of the spreadsheet.

- Step d is assumption of a starting value for the pipe diameter.

- Steps e, f, & g are the calculations in the column on the right in the upper part of the spreadsheet.

- Steps h and i are the iterative calculation of the Moody friction factor using the transition region equation, shown as step 2 in the spreadsheet.

- Steps j and k are the iterative calculation of the required pipe diameter, shown as step 3 in the spreadsheet.

The spreadsheet images above only show the final assumed and calculated values for **D**. In the original solution to this example, the first assumed value for **D** was 6 inches. Subsequent calculated and assumed values for **D**, leading to the final solution are as follows:

Assumed **D**, in	Calculated **D**, in.
6	0.2
4	1.0
3	3.4
3.5	1.8

This shows 3.5 inches to be the smallest standard pipe size that will be adequate. The next smaller size, 3 inch diameter, isn't large enough, because the calculated **D** is larger than 3 inches.

11.   Calculation of Pipe Flow Rate

Calculation of the fluid flow rate for a given fluid temperature through a pipe of known diameter, length, and material, with a specified allowable head loss can be done using the Darcy Weisbach equation. This is also an iterative process, because a value for the unknown fluid flow rate is needed to calculate the Reynolds number, which is needed to get a value for the friction factor. The step-by-step process for this calculation is as follows:

a. Determine the density, ρ, and viscosity, μ, of the flowing fluid at the specified temperature.

b. Obtain a value of pipe roughness, ε, for the specified pipe material.

c. Identify the known values for frictional head loss, h_L, pipe diameter, **D**, and pipe length, **L**.

d. Using the known values for **D** and ε, calculate the Moody friction factor, **f**, assuming completely turbulent flow.

e. Assume a starting value for the fluid flow rate, **Q**.

f. Calculate the fluid velocity, **V**.

g. Calculate the Reynolds number, **Re**

h. Use the calculated values of **Re** and **f** to recalculate **f** using the transition region equation (which gives **f** as a function of **Re** and **f**).

i. Use the following iterative process to calculate the Moody friction factor, **f**: If the value of **f** calculated in step **h** is different from that calculated in step d, repeat step h using the most recent value calculated for **f**. Repeat as many times as necessary until two subsequent calculations give the same value of **f**.

j. Using known values for h_L, **D**, and **L**, and the calculated value for **f**, calculate the average fluid velocity, **V**, with the Darcy Weisbach equation and calculate the fluid flow rate from $\mathbf{Q = V(\pi D^2/4)}$.

k. If this calculated value of **Q** is different than the assumed value, then replace the assumed Q value with the calculated value and carry out steps f through j to get another calculated value for **Q**. Repeat as many times as necessary to the calculated **Q** value to be the same as the assumed **Q** value. This iteration typically converges rapidly.

Example #9: Use an Excel spreadsheet to calculate the flow rate of water at 50°F, delivered through 40 ft of 4 inch diameter galvanized pipe, with a head loss of 0.9 ft.

Solution: The Excel spreadsheet in the image on the next page shows the solution to this example. The spreadsheet calculations correlate with the steps a through k outlined above for this type of calculation.

- Steps a, b, & c consist of entering values for the required inputs, h_L, ε, L, Q, ρ, and μ in the column on the left in the upper part of the spreadsheet.

- Step d is calculation of the Moody friction factor, **f**, assuming completely turbulent flow.

- Step e is assumption of a starting value for the fluid flow rate, **Q**.

- Steps f and g are calculation of the average fluid velocity, **V**, and then calculation of the Reynolds number, **Re**.

- Steps h and i are the iterative calculation of the Moody friction factor using the transition region equation, shown as step 2 in the spreadsheet.

- Steps j and k are the iterative calculation of the fluid flow rate, shown as step 3 in the spreadsheet.

Pipe Flow/Friction Factor Calculations II: (U.S. units)

Calculation of Fluid Flow Rate, Q, for given frictional head loss, h_L, pipe diam., D, pipe length, L, pipe roughness, ε, and fluid properties, ρ & μ.

Instructions: *Enter values in blue boxes. Spreadsheet calculates values in yellow boxes*

1. Determine Friction Factor, f, assuming completely turbulent flow $\{f = [1.14 + 2 \log_{10}(D/\varepsilon)]^{-2}\}$

Inputs (enter values in the blue boxes)

			Calculations		
Pipe Diameter, D_{in} =	4	in	Pipe Diameter, D =	0.3333	ft
Pipe Roughness, ε =	0.0005	ft	Friction Factor, f =	0.02170	
Pipe Length, L =	40	ft	Cross-Sect. Area, A =	0.0873	ft^2
Allowable Head Loss, h_L =	0.9	ft	Assumed Flow Rate, Q =	0.39	cfs
Fluid Density, ρ =	1.94	slugs/ft^3	Ave. Velocity, V =	4.5	ft/sec
Fluid Viscosity, μ =	0.0000273	lb-sec/ft^2	Reynolds number, Re =	105,861	

2. Check on whether the given flow is "completely turbulent flow"

(Calculate f with the transition region equation and see if differs from the one calculated above.)

$f = \{-2^*\log_{10}[((\varepsilon/D)/3.7)+(2.51/(Re^*(f^{1/2})))]\}^{-2}$

Transition Region Friction Factor, f:	f =	0.0237
Repeat calc of f using new value of f:	f =	0.0236
Repeat again if necessary:	f =	0.0236
Repeat again if necessary:	f =	0.0236

3. Calculate V and Q, using the final value for f calculated in step 2 in the Darcy Weisbach equation

Equations: V = $\{2g^*h_L/[f^*(L/D) + SK]\}^{1/2}$ and Q = $V(\pi D^2/4)$

Sum of minor loss coefficients, ΣK =	0		(set $\Sigma K = 0$ to calculate straight pipe losses)
Fluid Velocity, V =	4.5	ft/sec	
Fluid Flow Rate, Q =	0.39	cfs	

The spreadsheet image on the previous page only shows the final assumed and calculated values for **Q**. In the original solution to this example, the first assumed value for **Q** was 0.20 cfs. Subsequent calculated and assumed values for **Q**, leading to the final solution are as follows:

Assumed **Q**, cfs	Calculated **Q**, cfs
0.20	0.38
0.38	0.39
0.39	0.39

As shown in the table the iterative calculation converged rapidly to the solution: **Q = 0.39 cfs**.

12.   Calculations with S.I. Units

Use of S.I. units for pipe flow/friction factor/Darcy Weisbach calculations is rather straightforward. The Darcy Weisbach equation, the Moody diagram, and the Moody friction factor equations remain exactly the same.

The only change from calculations with U.S. units is use of mm instead of inches (or m instead of ft) for pipe diameter, **D**; m instead of ft for pipe length, **L**; m^3/s instead of cfs for fluid flow rate, **Q**; mm or m instead of ft for ε; kg/m^3 instead of $slugs/ft^3$ for density, ρ; $N\text{-}s/m^2$ instead of $lb\text{-}sec/ft^2$ for μ; and m instead of ft for frictional head loss, **h_L**. The following example illustrates S.I. calculations.

Example #10:   Calculate the frictional head loss in m and the frictional pressure drop in kN/m^2, for a flow rate of 0.017 m^3/s of water at 10^oC, through a 30 m length of 150 mm diameter wrought iron pipe, using an Excel spreadsheet and the appropriate Moody friction factor equation(s).

Solution: The steps in the solution are the same as those given above for Example #5 (a similar problem in U.S. units). The spreadsheet image from a similar spreadsheet set up for S.I. calculations on the next page shows the solution.

Pipe Flow/Friction Factor Calculations I: (S.I. units)

Calculation of Head Loss, h_L, and Frictional Pressure Drop, ΔP_f,

for given flow rate, Q, pipe diam., D, pipe length, L,

pipe roughness, ε, and fluid properties, ρ & μ.

Instructions: *Enter values in blue boxes. Spreadsheet calculates values in yellow boxes*

1. Determine Friction Factor, f, assuming completely turbulent flow $\{f = [1.14 + 2\log_{10}(D/\varepsilon)]^{-2}\}$

Inputs (enter values in the blue boxes)   **Calculations** (done by spreadsheet)

Inputs			Calculations		
Pipe Diameter, D_{in} =	150	mm	Pipe Diameter, **D** =	0.1500	m
Pipe Roughness, ε =	0.15	mm	Friction Factor, **f** =	0.01962	
Pipe Length, **L** =	30	m	Cross-Sect. Area, **A** =	0.0177	m^2
Pipe Flow Rate, **Q** =	0.017	m^3/s	Ave. Velocity, **V** =	1.0	m/s
Fluid Density, ρ =	1000	kg/m^3	Reynolds number, **Re** =	111,000	
Fluid Viscosity, μ =	0.0013	$N\text{-}s/m^2$			

2. Check on whether the given flow is "completely turbulent flow"

(Calculate f with the transition region equation and see if differs from the one calculated above.)

$f = \{-2\log_{10}[((\varepsilon/D)/3.7)+(2.51/(Re*(f^{1/2})))]\}^{-2}$

Transition Region Friction Factor, f:	f =	0.0221
Repeat calc of f using new value of f:	f =	0.0220
Repeat again if necessary:	f =	0.0220

3. Calculate h_L and ΔP_f, using the final value for f calculated in step 2

$(h_L = f(L/D)(V^2/2g)$ and $\Delta P_f = \rho g h_L)$

Frictional Head Loss, h_L =	0.207	m
Frictional Pressure Drop, ΔP_f =	2033	N/m^2
Frictional Pressure Drop, ΔP_f =	2.03	kN/m^2

The first part of the spreadsheet solution is entering the known input values in the upper left column. Then the spreadsheet calculations will proceed, leading to a value for the Moody frictions factor based on completely turbulent flow and a preliminary value for Reynolds number.

The second part of the spreadsheet solution is an iterative solution to calculate the Moody friction factor using the transition region equation.

The third and final part of the spreadsheet solution is calculation of frictional head loss and frictional pressure drop using the Darcy Weisbach equation.

The spreadsheet shows the final solution:
$h_L = 0.207$ **m** and $\Delta P_f = 2.03$ **kN/m^2**.

Calculation of required pipe diameter using S.I. units will be essentially the same as the calculations in Example #7 and calculation of fluid flow rate with S.I. units will be essentially the same as the calculations in Example #8.

13.   Summary

The Darcy Weisbach equation and the Moody friction factor equations given in this course are the essential tools for calculations involving the parameters fluid flow rate, **Q**, through a pipe of diameter, **D**, length, **L**, and roughness, ε, with frictional head loss, h_L. Other parameters that are often used in the equations are: fluid density, ρ, fluid viscosity, μ, average fluid velocity, **V**, and Reynolds number, **Re**.

This course included discussion of the steps used in three common types of calculations with these parameters that require one or more iterative calculations. The three types of calculations are calculation of frictional head loss, required pipe diameter, or fluid flow rate, when the other parameters are known. The use of Excel spreadsheets for the calculations was illustrated with several examples.

14.   References and Websites

1. Moody, L.F., "Friction Factors for Pipe Flow," Trans ASME, pp 671-684, November, 1944.

2. Munson, B. R., Young, D. F., & Okiishi, T. H., Fundamentals of Fluid Mechanics, 4th Ed., New York: John Wiley and Sons, Inc, 2002.

3. Darcy Weisbach equation history -
http://biosystems.okstate.edu/darcy/DarcyWeisbach/Darcy-WeisbachHistory.htm

4. U.S. EPA, *EPANet2 User Manual,*
http://www.epa.gov/nrmrl/wswrd/dw/epanet/EN2manual.PDF